4.3 Investigate Transformations of Quadratics

When police investigate car accidents, they often measure the length of a tire skid mark to determine the speed of the car before braking. Such calculations involve quadratic relations. On dry pavement, the length of a skid mark, l, is related to the speed of the car, s, before braking by the relation $l = 0.04s^2$.

Investigate

How do transformations of the graph of $y = x^2$ affect the equation?

Tools
- TI-83 Plus or TI-84 Plus graphing calculator
- grid paper

A: Compare the Graphs of $y = x^2$ and $y = x^2 + k$

1. First, clear any graphed equations.
   - Press [Y=] and use the [CLEAR] key to remove any equations.
   - Make sure **Plot1**, **Plot2**, and **Plot3** are not highlighted. If they are, use the [▸], [◂], [▴], and [▾] keys to move to each and press [ENTER].

2. Use a standard window.
   - Press [ZOOM] and select **6:ZStandard.**
   - You can view the window settings by pressing [WINDOW].

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WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

3. Graph the equation $y = x^2$ as **Y1**.
   - Press [Y=]. Beside **Y1=**, press [X,T,θ,n][x²].
   - Press [GRAPH] to view the parabola.

4. Enter $y = x^2 + 2$ as **Y2** and $y = x^2 - 4$ as **Y3**. Press [GRAPH].

5. **a)** Sketch all three graphs on the same set of axes. Label each parabola with its equation.
   b) Describe the transformations.
   c) Without using a graphing calculator, sketch the graph of $y = x^2 - 8$.

6. Reflect Describe how the value of k in $y = x^2 + k$ changes the graph of $y = x^2$.

Technology Tip

A table of values can help you sketch the graph of an equation entered using (Y=). You can specify how a table of values is set up.

- Press (2nd) [TBLSET] to display the **TABLE SETUP** screen. Make sure both **Indpnt** and **Depend** are set to **Auto**. Enter the desired starting x-value (**TblStart**) and x increment (**ΔTbl**). For example, try **TblStart=-10** and **ΔTbl=1**.
- Press (2nd) [TABLE] to view the table of values.

B: Compare the Graphs of $y = x^2$ and $y = ax^2$

1. **a)** Clear the equations from **Y2=** and **Y3=**, but keep $y = x^2$ in **Y1**.
   b) Graph the equations $y = 2x^2$ and $y = 3x^2$.

2. **a)** Sketch all three graphs on the same set of axes. Label each parabola with its equation.
   b) Describe the transformations.

3. Without using a graphing calculator, sketch the graph of $y = 4x^2$.

4. Repeat steps 1 and 2 for the equations $y = \frac{1}{2}x^2$ and $y = \frac{1}{4}x^2$.

5. Without using a graphing calculator, sketch the graph of $y = \frac{1}{3}x^2$.

6. Repeat steps 1 and 2 for the equations $y = -2x^2$ and $y = -0.5x^2$.

7. Without using a graphing calculator, sketch the graph of $y = -3x^2$.

8. Reflect Describe how the value of a in $y = ax^2$ changes the graph of $y = x^2$.

C: Compare the Graphs of $y = x^2$ and $y = (x - h)^2$

1. **a)** Clear all equations except $y = x^2$.
   b) Graph the equations $y = (x - 2)^2$ and $y = (x - 5)^2$.

2. **a)** Sketch all three graphs on the same set of axes. Label each parabola with its equation.
   b) Describe the transformations.

3. Without using a graphing calculator, sketch the graph of $y = (x - 3)^2$.

4. Repeat steps 1 and 2 using the equations $y = (x + 2)^2$ and $y = (x + 5)^2$.

5. Without using a graphing calculator, sketch the graph of $y = (x + 3)^2$.

6. Reflect Describe how the value of h in $y = (x - h)^2$ changes the graph of $y = x^2$.

Example Falling Stone

A stone is dropped from the top of a 50-m cliff above a river. Its height, y, in metres, above the water can be estimated using the relation $y = -4.9x^2 + 50$, where x is the time, in seconds.

a) Graph the relation.

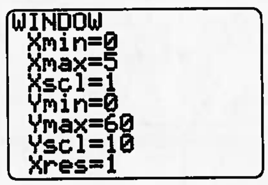
b) Find the intercepts. What do they represent?

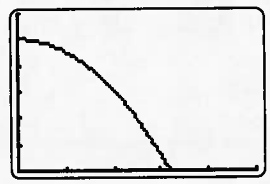
c) How would the equation change if the stone were dropped from a 75-m cliff instead of a 50-m cliff?

d) For what values of x is each equation valid?

Solution

a) Use a graphing calculator with the window settings shown.





Since height and time cannot be negative, this graph shows only part of a parabola.

b) For the y-intercept, let $x = 0$.

$$y = -4.9(0)^2 + 50$$
$$= 50$$

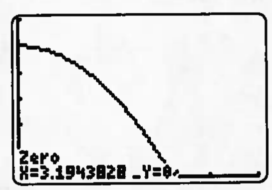
I can see that the y-intercept is 50 from the graph.

The y-intercept is 50. This represents the height from which the stone was dropped, 50 m above the water.

zero
- a value of x for which a relation has a value of 0
- corresponds to an x-intercept of the graph of the relation

To find the x-intercept, or **zero**, of the relation use the Zero operation on a graphing calculator.

- Press 2nd [CALC] to display the **CALCULATE** menu, and select **2:zero**.
- Move the cursor to the left of the x-intercept and press ENTER.
- Move the cursor to the right of the x-intercept and press ENTER.
- Press ENTER again.

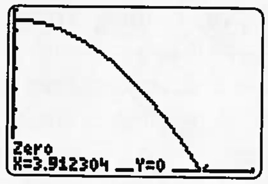


The x-intercept is approximately 3.19.
This represents the time when the stone hits the water, 3.19 s.

c) The constant term would be 75 instead of 50. The equation would change to $y = -4.9x^2 + 75$.

d) The original equation $y = -4.9x^2 + 50$ is valid for $0 \le x \le 3.19$ (approximately).

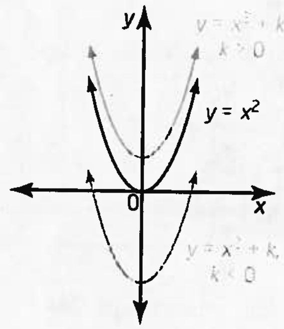
Graph the new equation and use the Zero operation. You will need to change the WINDOW settings to be able to see the whole graph.



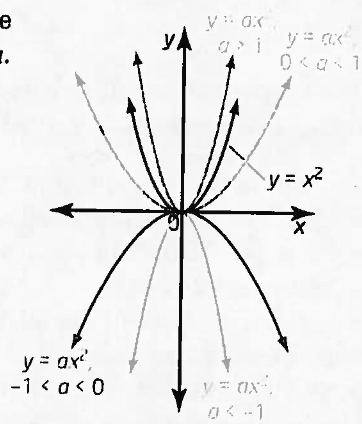
The new equation $y = -4.9x^2 + 75$ is valid for $0 \le x \le 3.91$ (approximately).

Key Concepts

- To graph $y = x^2 + k$, translate the graph of $y = x^2$ vertically k units.
  - If $k > 0$, then the graph is translated k units upward.
  - If $k < 0$, then the graph is translated k units downward.



- To graph $y = ax^2$, stretch or compress the graph of $y = x^2$ vertically by a factor of a.
  - If $a < 0$, the parabola is reflected in the x-axis.
  - If $a > 1$ or $a < -1$, then the graph is stretched vertically (narrows).
  - If $-1 < a < 0$ or $0 < a < 1$, then the graph is compressed vertically (widens).



- To graph $y = (x - h)^2$, translate the graph of $y = x^2$ horizontally h units.
  - If $h > 0$, then the graph is translated h units to the right.
  - If $h < 0$, then the graph is translated h units to the left.